Extensive form games

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the general rate of profits (if corn is a means of production), an increase in the price of the agricultural commodity in relation the industrial ones and a consequent rise in rents on the more fertile lands. In a similar way, in the case of intensive rent, an increase in the quantity produced of a certain agricultural good will cause a change in the methods of production – the old pair of techniques will give place to a new more efficient one – with a consequent increase in the agricultural price and rent, and the reduction of the general rate of profits. Both results (Montani, 1972) are perfectly in agreement with the Ricardian doctrine of rent.

(b) The order of fertility of the various kinds of land is not given, once and for all, by nature. The more fertile lands (i.e. lands which are put into cultivation first because of the rate of profits they give to the agricultural entrepreneur) do not coincide with the lands paying higher rents. Ricardo's opinion is not correct on this point:

> When land of the third quality is taken into cultivation, rent immediately commences on the second. At the same time, the rent of the first quality will rise, *for that must always be above the rent of the second*, by the difference between the produce which they yield with a given quantity of capital and labour (Ricardo, p. 70; our italics).

Generally speaking, if lands 1, 2 and 3 are already cultivated and rent on land 1 is higher than rent on land 2, it may happen that when land 4 is put into cultivation the rate of rent on land 2 may become greater than the rate of rent on land 1. The reversal of the order of rents is possible both for the rate of rent per unit of land and for rate of rent per unit of product (Montani, 1972).

(c) The scarcity of land does not depend only on the quantity of the agricultural product to be produced, but on the distribution of income between profits and wages too. Given certain methods of production, some natural resources, as land, mineral deposits, etc, become 'scarce' or 'redundant' according to the quantity produced of a given commodity and to the relative level of the rate of profits in relation to wages. This may happen both for extensive and intensive rent. Therefore, since no change in the proportions of the 'production factors' occurs when the natural resource becomes scarce or redundant owing to the change in distribution, it is obvious that the meaning of 'decreasing returns' inside the classical theory of rent (and distribution) is different from that connected with the 'law of variable proportions' (Montani, 1975).

The vicissitudes of the theory of rent in the history of economic thought are, therefore, strictly connected to the meaning of the law of decreasing returns; this can be appreciated from what has been said above about how the content of this law changed considerably during the transition from the classical to the marginalist paradigm. Marshall was well aware of the diversity of the two points of view, and stated quite clearly that

> the diminishing return which arises from an ill-proportioned application of the various agents of production into a particular task has little in common with that broad tendency to the pressure of a crowded and growing population on the means of subsistence. The great classical Law of Diminishing Return has its chief application, not to any one particular crop, but to all the chief food crops (Marshall, p. 338).

This classical meaning of the law was progressively forgotten by the economists owing to the over-narrow view imposed by the marginalist theory of distribution.

GUIDO MONTANI

See also ABSOLUTE RENT; LAND RENT; RENT; RICARDO, DAVID.

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extensive form games. The most general model used to describe conflict situations is the extensive form model, which specifies in detail the dynamic evolution of each situation and thus provides an exact description of 'who knows what when' and 'what is the consequence of which'. The model should contain all relevant aspects of the situation; in particular, any possibility of (pre)commitment should be explicitly included. This implies that the game should be analysed by solution concepts from noncooperative game theory, that is, refinements of Nash equilibria. The term extensive form game was coined in von Neumann and Morgenstern (1944) in which a set theoretic approach was used. We will describe the graph theoretical representation proposed in Kuhn (1953) that has become the standard model. For convenience, attention will be restricted to finite games.

The basic element in the Kuhn representation of an n-person extensive form game is a rooted tree, that is, a directed acyclic graph with a distinguished vertex. The game starts at the root of the tree. The tree's terminal nodes correspond to the endpoints of the game and associated with each of these there is an n-vector of real numbers specifying the payoff to each player (in von Neumann–Morgenstern utilities) that results from that play. The nonterminal nodes represent the decision points in the game. Each such point is labelled with an index i ($i \in \{0, 1, \ldots, n\}$) indicating which player has to move at that point. Player O is the chance player who performs the moves of nature. A maximal set of decision points that a player cannot distinguish between is called an information set. A choice at an information set associates a unique successor to every decision point in this set, hence, a choice consists of a set of edges, exactly one edge emanating from each point in the set.

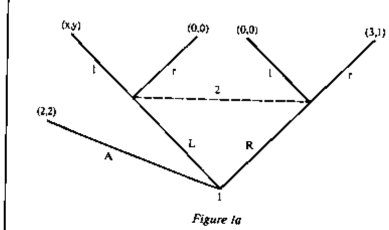

Figure 1a

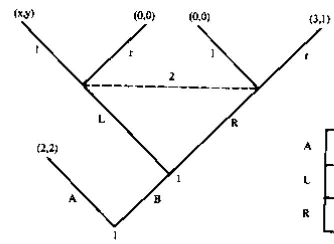

Figure 1b

	l	r
A	2,2	2,2
L	x,y	0,0
R	0,0	3,1

Figure 1c

Information sets of the chance player are singletons and the probability of each choice of chance is specified. Formally then, an extensive form game is a sixtuple $\Gamma = (K, P, U, C, p, h)$ which respectively specify the underlying tree, the player labelling, the information sets, the choices, the probabilities of chance choices and the payoffs.

As an example, consider the 2-person game of Figure 1a. First player 1 has to move. If he chooses A, the game terminates with both players receiving 2. If he chooses L or R, player 2 has to move and, when he is called to move, this player does not know whether L or R has been chosen. Hence, the 2 decision points of player 2 constitute an information set and this is indicated by a dashed line connecting the points. If the choices L and ℓ are taken, then player 1 receives x, while player 2 gets y. The payoff vectors at the other endpoints are listed similarly, that is, with player 1's payoff first. The game of Figure 1b differs from that in Figure 1a only in the fact that now player 1 has to choose between L and R only after he has decided not to choose A. In this case, the game admits a proper subgame starting at the second decision point of player 1. This subgame can also be interpreted as the players making their choices simultaneously.

A strategy is a complete specification of how a player intends to play in every contingency that might arise. It can be planned in advance and can be given to an agent (or a computing machine) who can then play on behalf of the player. A pure strategy specifies a single choice at each information set, a behaviour strategy prescribes local randomization among choices at information sets and a mixed strategy requires a player to randomize several pure strategies at the beginning of the game. The normal form of an extensive game is a table listing all pure strategy combinations and the payoff vectors resulting from them. Figure 1c displays the normal form of Figure 1a, and, up to inessential details, this also represents the game of Figure 1b. The normal form suppresses the dynamic structure of the extensive game and condenses all decision-making into one stage. This normalization offers a major conceptual simplification, at the expense of computational complexity: the set of strategies may be so large that normalization is not practical. Below we return to the question of whether essential information is destroyed when a game is normalized.

A game is said to be of perfect recall if each player always remembers what he has previously known or done, that is, if information is increasing over time. A game may fail to have perfect recall when a player is a team such as in bridge and in this case behaviour strategies may be inferior to mixed strategies since the latter allow for complete correlation between different agents of the team. However, by modelling different agents as different players with the same payoff function one can restore perfect recall, hence, in the literature attention is usually restricted to this class of games. In Kuhn (1953) and Aumann (1964) is has been shown that, if there is perfect recall, the restriction to behaviour strategies is justified.

A game is said to be of perfect information if all information sets are singletons, that is, if there are no simultaneous moves and if each player always is perfectly informed about anything that happened in the past. In this case, there is no need to randomize and the game can be solved by working backwards from the end (as already observed in Zermelo, 1913). For generic games, this procedure yields a unique solution which is also the solution obtained by iterative elimination of dominated strategies in the normal form. The assumption of the model that there are no external commitment possibilities implies that only this dynamic programming solution is viable; however, this generally is not the unique Nash equilibrium. In the game of Figure 2, the roll-back procedure yields (R, r), but a second equilibrium is (L, ℓ). The latter is a Nash equilibrium since player 2 does not have to execute the threat when it is believed. However, the threat is not credible: player 2 has to move only when 1 has chosen R and facing the *fait accompli* that R has been chosen, player 2 is better off choosing r. Note that it is essential that 2 cannot commit himself: If he could we would have a different game of which the outcome could perfectly well be $(2, 2)$.

A major part of noncooperative game theory is concerned with how to extend the backwards induction principle to games with imperfect information, that is how to exclude intuitively

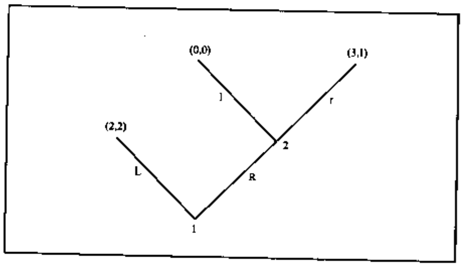

Figure 2

unreasonable Nash equilibria in general. This research originates with Selten (1965) in which the concept of subgame perfect equilibria was introduced, that is, of strategies that constitute an equilibrium in every subgame. If $y < 0$, then the unique equilibrium of the subgame in Figure 1b is (R, r) and, consequently, (BR, r) is the unique subgame perfect equilibrium in that case. If $x < 2$, however, then (AL, ℓ) is an equilibrium that is not subgame perfect. The game of Figure 1a does not admit any proper subgames; hence, any equilibrium is subgame perfect, in particular (A, ℓ) is subgame perfect if $x > 2$. This shows that the set of subgame perfect equilibria depends on the details of the tree and that the criterion of subgame perfection does not eliminate all intuitively unreasonable equilibria.

To remedy the latter drawback, the concept of (trembling hand) perfectness was introduced in Selten (1975). The idea behind this concept is that with a small probability players make mistakes, so that each choice is taken with an infinitesimal probability and, hence, each information set can be reached. If $y \leqslant 0$, then the unique perfect equilibrium outcome in Figures 1a and 1b is (1, 3): player 2 is forced to choose r since L and R occur with positive probability.

The perfectness concept is closely related to the sequential equilibrium concept proposed in Kreps and Wilson (1982). The latter is based on the idea of 'Bayesian' players who construct subjective beliefs about where they are in the tree when an information set is reached unexpectedly and who maximize expected payoffs associated with such beliefs. The requirements that beliefs be shared by players and that they be consistent with the strategies being played (Bayesian updating) imply that the difference from perfection is only marginal. In Figure 1a, only (R, r) is sequential when $y < 0$. When $y = 0$, then (A, ℓ) is sequential, but not perfect: choosing ℓ is justified if one assigns probability 1 to the mistake L, but according to perfectness R also occurs with a positive probability.

Unfortunately, the great freedom that one has in constructing beliefs implies that many intuitively unreasonable equilibria are sequential. In Figure 1a, if $y > 0$, then player 2 can justify playing ℓ by assigning probability 1 to the 'mistake' L, hence, (A, ℓ) is a sequential equilibrium if $x \leqslant 2$. However, if $x < 0$, then L is dominated by both A and R and thinking that 1 has chosen L is certainly nonsensical. (Note that, if $x < 0$, then (AL, ℓ) is not a sequential equilibrium of the game of Figure 1b, hence, the set of sequential (perfect) equilibrium outcomes depends on the details of the tree.) By assuming that a player will make more costly mistakes with a much smaller probability than less costly ones (as in Myerson's (1978) concept of proper equilibria) one can eliminate the equilibrium (A, ℓ) when $x \leqslant 0$ (since then L is dominated by R), but this does not work if $x > 0$. Still, the equilibrium (A, ℓ) is nonsensical if $x < 2$: If player 2 is reached, he should conclude that player 1 has passed off a payoff of 2 and, hence, that he aims for the payoff of 3 and has chosen R. Consequently, player 2 should respond by r: only the equilibrium (R, r) is viable.

What distinguishes the equilibrium (R,r) in Figure 1 is that this is the only one that is stable against all small perturbations of the equilibrium strategies, and the above discussion suggests that such equilibria might be the proper objects to study. An investigation of these stable equilibria has been performed in Kohlberg and Mertens (1984) and they have shown that whether an equilibrium outcome is stable or not can already be detected in the normal form. This brings us back to the question of whether an extensive form is adequately represented by its normal form, that is, whether two extensive games with the same normal form are equivalent. One answer is that this depends on the solution concept employed: it is affirmative for Nash equilibria, for proper equilibria (van Damme, 1983, 1984) and for stable equilibria, i.e. for the strongest and the weakest concepts, but it is negative for the intermediate concepts of (subgame) perfect and sequential equilibria. A more satisfactory answer is provided by a theorem of Thompson (1952) (see Kohlberg and Mertens, 1984) that completely characterizes the class of transformations that can be applied to an extensive form game without changing its (reduced) normal form: The normal form is an adequate representation if and only if these transformations are inessential. Nevertheless, the normal form should be used with care, especially in games with incomplete information (cf. Harsanyi, 1967–8; Aumann and Maschler, 1972), or when communication is possible (cf. Myerson, 1986).

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See also COOPERATIVE GAMES; GAMES WITH INCOMPLETE INFORMATION; GAME THEORY; NASH EQUILIBRIUM; NON-COOPERATIVE GAMES.

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external debt. The term 'external debt' refers to financial obligations incurred by individuals or, more commonly, institutions resident in one country vis-à-vis those resident in another. In other words, the obligations cross the borders of sovereign states. Usually different nationalities or citizenships are involved, as well as different residencies, but this is not strictly necessary. For instance, a US corporation in the